

Date: Tue, 04 Apr 2000 19:39:00 -0400 (EDT) From: Graham Solomon
<gsolomon@wlu.ca>
Subject: fulfillability 2
To: Allen Hazen <a.hazen@philosophy.unimelb.edu.au> Reply-to: Graham
Solomon <gsolomon@wlu.ca> MIME-version: 1.0

Proof of THEOREM:

(1) 'T is fulfilled' is true in N

First Problem: how to show of an infinite set of sentences that each sentence is fulfilled?

Suppose T is a RE set = $\phi_1, \phi_2, \dots, \phi_n \dots$. Then we know by Craig's theorem that

ϕ_1

$\phi_1 \& \phi_2$

.

.

$\phi_1 \& \phi_2 \& \dots \& \phi_n$

.

.

is a recursive set of axioms for T.

Let us define a sequence of sentences

$T_1 = \phi_1$

.

.

$T_{n+1} = T_n \& \phi_{n+1}$

Clearly, if we can show $(\forall n)(T_n \text{ is } n\text{-fulfilled})$ we will have shown that T is fulfilled. For suppose T is not fulfilled. Then for some n, T is not n-fulfilled. So some sentence of T is not fulfilled by a good sequence of length n, for some n. But then this sentence conjoined with any number of others is not n-fulfilled (FACT 4). Choose a T_n containing this sentence with n big enough!

Thus we show $(\forall n)(T_n \text{ is } n\text{-fulfilled})$

We show for arbitrary n that T_n is n-fulfilled. By FACT 3 PAI- $T_n \rightarrow T_n$ is n-fulfilled So TI- $T_n \rightarrow T_n$ is n-fulfilled

TI- T_n is n-fulfilled

But T is w-consistent i.e. omega-consistent {see * below 'missing lemma?}

So, T notI- $(\exists n)(T_n \text{ is not } n\text{-fulfilled})$ Thus we have $N \models T_n$ is n-fulfilled

{possible subtlety here: Add T's soundness with respect to N to the hypothesis of THEOREM ?}

Note if T were not w-consistent we could have $T \models T_n$ is n-fulfilled, and $T \models (\exists n)(T_n \text{ is not n-fulfilled})$

Thus any model of T would be such that for some a in M,
 $M \models T_a$ is not a-fulfilled
while for arbitrary n
 $M \models T_n$ is n-fulfilled

(Clearly a is a non-standard element)

w-consistency is used to show that there is a model viz N of T such that for arbitrary n in N
 $N \models T_n$ is n-fulfilled
Thus justifying
 $N \models (\forall n)(T_n \text{ is n-fulfilled})$
Hence
 $N \models T$ is fulfilled

{**missing lemma?**} For w-consistent (consistent with PA) extensions of PA, w-consistency implies truth in N }

(2) Construction of our nonstandard model M

Take any equivalent model with nonstandard part. Then for each n in \mathbb{N}^+ , for each standard number,
 $\models T_n$ is n-fulfilled
by our previous argument

So for some v in ${}^*\mathbb{N}$ (in the nonstandard part of our model)
 $\models T_v$ is v-fulfilled

{Note: T_v cannot be well ordered. Something like this must be true,
 $T_v = \phi_1 \ \& \ \phi_2 \ \& \ \dots \ \& \ \phi_n \ \& \ \dots; \dots \ \& \ \phi_v$. But this isn't clear to me}

So there is a s in ${}^*\mathbb{N}$ such that $l(s) = v$ and s is good and s fulfills T_v

Notice that $\forall n \in \mathbb{N}^+, \models s$ fulfills T_n

Let s_v be the least such s. Let s' be the first w-terms of s_v . Let M be the part of ${}^*\mathbb{N}$ cofinal with s' : $x \in M$ iff $x \in {}^*\mathbb{N} \ \& \ (\exists i \in \mathbb{N}^+)(x \leq s(i))$

Remark: Notice v in M because $s(1) > v$ by proposition (i) of good sequences [$v = l(s)$] (Of course M contains many nonstandard numbers)

(3) $M \models T$

Proof: $*I \models s_v$ fulfills T_v for all v

So $M \models s'$ fulfills T_v for all v

So $M \models T_v$ for all v because s' is an w -sequence cofinal with M (FACT 1)

(4) $M \not\models T$ is fulfilled

Proof: suppose $M \models T$ is fulfilled

then for some s in M , $M \models l(s) = v$ & s is good & s fulfills T_v .

(Recall v is in M) But s_v is the least such s in $*N$ and s_v is not in M .

END OF NOTES